Can you read this ?

Chapter 2

Time series regression

$$y_t = \beta_0 + \beta_1 x_{1t} + \cdots + \beta_q x_{qt} + w_t$$

where $w_t \sim WN(\sigma_w^2)$

Usual results of regression apply.
If w_t is not white noise, estimates are consistent but standard errors are wrong.

- autocorrelation
- heteroskedasticity

Example: Estimating a time trend

Assessing model fit

- Akaike Information Criterion (AIC)

$$AIC = \log \hat{\sigma}^2 + \frac{n + 2k}{n}$$

or $$AIC = -2 \log L + 2k$$

where k is the number estimated model parameters.

Prefer model with minimum AIC.

- Bias corrected AIC

$$AIC_c = \log \hat{\sigma}^2 + \frac{\overset{2k??}{n+k}}{n-k-2}$$

- Bayesian Information Criterion (BIC) also known as Schwarz Criterion

$$BIC = \log \hat{\sigma}^2 + \frac{k \log N}{n}$$

or

$$BIC = -2 \log L + k \left(\log(n) - \log(2\pi) \right)$$

2.2 Exploratory data analysis

In order to do anything useful, we would like to have a stationary process.

Example: stationary about a time trend

$$x_t = \mu_t + y_t$$

where y_t is stationary.

Typically, might estimate μ_t and work

$$\hat{y}_t = x_t - \hat{\mu}_t.$$

Another way to get stationarity is to difference the data.

given $\{x_t\}$, construct

$$y_t = x_t - x_{t-1}$$
$$= \nabla x_t$$

Differencing

Theorem: If x_t is stationary, then so is ∇x_t.

(differencing preserves stationarity)

proof: $E[\nabla x_t] = E[x_t - x_{t-1}] = E[x_t] - E[x_{t-1}] = \mu_x - \mu_x$
$= 0$

$$\mathrm{Cov}(\nabla x_t, \nabla x_s) = E[(x_t - x_{t-1})(x_s - x_{s-1})]$$

$$= \mathrm{cov}(x_t, x_s) - \mathrm{cov}(x_t, x_{s-1}) - \mathrm{cov}(x_{t-1}, x_s) + \mathrm{cov}(x_{t-1}, x_{s-1})$$

let $h = s-t$

$$= \gamma_x(h) - \gamma_x(h-1) - \gamma_x(h+1) + \gamma_x(h)$$

$$= 2\gamma_x(h) - \gamma_x(h-1) - \gamma_x(h+1)$$

$E[\nabla x_t]$ is constant & $\mathrm{cov}(\nabla x_t, \nabla x_s)$ depends only on $s-t$. So stationary.