

A time-series is a sequence of r.v.'s x_1, x_2, x_3 where x_t indicates the value taken by the sequence at time t .

White noise: $\{w_t\}$ where $w_t \sim WN(0, \sigma_w^2)$ is uncorrelated with mean 0 & variance σ_w^2 .

white independent noise means the w_t are independent.

Gaussian white noise is where the w_t are all $N(0, \sigma_w^2)$.

Gaussian implies independent WN process, since

X & Y are uncorrelated if

$$E[(X - \mu_x)(Y - \mu_y)] = 0$$

$$\text{cov}(X, Y) = 0$$

X & Y are independent if

Let A & B be sets

$$P(X \in A \text{ and } Y \in B) = P(X \in A) \cdot P(Y \in B)$$

Exercise 1: generate a realization from
a Gaussian white noise process with
 $\sigma_w^2 = 1$. Plot it.

↳ Moving averages & filtering.

Example:

$$\text{let. } w_t \sim \text{GWN}(0, 1) \quad t = 1, \dots, 100$$

$$\text{let } v_t = \frac{1}{3}(w_{t-1} + w_t + w_{t+1}) \quad t = 2 \dots 99$$

In general,

$$\hookrightarrow v_t = \beta_{t-k} w_{t-k} + \beta_{t-k+1} w_{t-k+1} + \dots +$$

$$\beta_{t+k-1} w_{t+k-1} + \beta_{t+k} w_{t+k}$$

$$= \sum_{s=t-k}^{t+k} \beta_s w_s$$

one sided means $\beta_{t-k} = \dots = \beta_{t-1} = 0$ or

$$\beta_{t+1} = \dots = \beta_{t+k} = 0$$