3.7 : Integrated models

$$\nabla y_t = y_t - y_{t-1}$$

$$\nabla^d y_t = (1-B)^d y_t$$

$\{y_t\}$ is said to be ARIMA(p,d,q) if $\nabla^d y_t$ is ARMA(p,q).

We can write the model

$$\phi(B)(1-B)^d y_t = \theta(B) w_t$$

Diagnostics

Typically desirable to have residuals that look like white noise. Otherwise we may wish to do more modelling work.

ACF gives a graphical tool.

Ljung-Box (similar to Box-Pierce)

$$Q = n\cdot(n+2)\sum_{h=1}^{H} \frac{\hat{\rho}_e^{\,2}(h)}{n-h}.$$

Under H_0: residuals are white noise

$$Q \sim \chi^2_{H-p-q}.$$

In R, use the Box.test() function

Sometimes people also test for normality of residuals.

A graphical diagnostic is the normal-quantile plot

Given $\{\hat{w}_t\}$, let

$$Z_t = \Phi^{-1}\left(\frac{t}{T+1}\right) \qquad t = 1, 2, \ldots T$$

where Φ is the normal ~~pdf~~ cdf

Then plot (Z_t, r_t), where r_t = sorted $\hat{w}_t$

If residuals are normal these points should be close to a diagonal line through the origin. If $\sigma_w^2 = 1$, it will be the 45° line.

Fat tailed residuals are indicated by points below the diagonal line in the left tail & above the diagonal in the right tail.

A commonly used statistical test for normality is the Jarque-Bera test.

jarque.bera.test() in tseries package.

3.9. Regression with autocorrelated errors

$$y_t = \beta_0 + \beta_1 z_{t1} + \cdots + \beta_r z_{tr} + \varepsilon_t$$

where $\{\varepsilon_t\}$ are autocorrelated.

I might try our usual tools on the $\{\hat{\varepsilon}_t\}$.

* Typing...