Econ 522 Midterm, part 1 (Spring 2020)

Do the following problems using pencil and paper. Scan and submit on polylearn.

1. Consider the time series
$$y_t = \delta + y_{t-1} + w_t, \qquad w_t \sim WN(\sigma_W^2)$$
with initial condition $y_0 = 0$. Is $\{y_t\}$ stationary? What is the mean function, $\mu_y(t)$? What is the autocovariance function, $\gamma_y(s,t)$?

2. Consider the time series
$$x_t = \phi x_{t-1} + w_t$$
$$y_t = \alpha + w_t + u_t + \theta u_{t-1}$$
where $|\phi| < 1$ and $\{w_t\}$ and $\{u_t\}$ are independent white noise processes with variance σ_w^2 and σ_u^2 respectively.
   (a) What is $\gamma_x(h)$, the autocovariance function of $\{x_t\}$?
   (b) What is $\gamma_y(h)$, the autocovariance function of $\{y_t\}$?
   (c) What is $\gamma_{xy}(h)$, the crosscovariance function of $\{x_t\}$ and $\{y_t\}$?

3. Consider the model
$$y_t = \alpha - 1.7y_{t-1} + 0.6y_{t-2} + w_t + 0.4w_{t-1},$$
where w_t is white noise with variance σ_w^2. Is this model causal? Is it invertible? (Why?)

4. Consider the model
$$y_t = 0.3y_{t-1} + w_t + 1.5w_{t-1},$$
where w_t is white noise with variance σ^2.
Write this model in form $y_t = \sum_{i=0}^{\infty} \psi_i w_{t-i}$. (Show how to compute ψ_i, $i = 1, 2, \ldots$, in terms of the model coefficients.)